# Characterization of Tracer Responses Using Fractional Derivative-Based Mathematical Model and Its Application to Prediction of Mass Transport in Fractured Reservoirs

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***Keywords***

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## ABSTRACT

Tracer testing is a standard method for tracing mass transport within a geothermal reservoir and can be a valuable tool in the design and management of production and injection operations. In this study, we discuss the use of fractional advection-dispersion equation (fADE) model to characterize tracer responses with the objective of predicting mass transport in complex fractured reservoirs. A 3D fracture network model for flow analysis (FRACSIM-3D) is utilized to produce numerical data of tracer responses in fractured reservoirs. It has been shown that the FRACSIM-3D reproduces highly anomalous behavior of mass transport, which is attributable to the preferential pathways that arise because of the degree of fracture connectivity. The fADE mathematical model is applied to analyze the numerical tracer results simulated by FRACSIM-3D. For comparison, the advection-dispersion equation (ADE) is also used to characterize the tracer responses in addition to the fADE. Based on the tracer data obtained for a well interval of 50m, both the fADE and ADE model are applied to predict the tracer responses in the case where the well spacing is extended to 80m. It is demonstrated that the ADE model produces tracer curves which deviate significantly from the FRACSIM-3D results particularly for long-term behaviors, while the tracer responses predicted by the fADE model are in reasonable agreement with the numerically obtained data by FRACSIM-3D.

## 1. Introduction

Reinjection is an important problem in all geothermal fields to prolong the life of the power station. The reinjected water is reheated as it passes hot reservoir rocks while moving from injection wells back to production wells, and can improve productivity by increasing reservoir pressures and replacing produced reservoir fluids. In several theoretical and practical studies, it has been shown that reinjection is a powerful method for increasing the longevity of geothermal resources and the amount of energy that can be extracted from a given reservoir (Horne,1985; Stefansson,1997; Kaya et al., 2011). However, in highly fractured reservoirs, the thermal energy in the rocks along the path of the reinjected water becomes depleted rapidly, and the enthalpy of the water arriving at the production wells begins to drop. Therefore, the obvious engineering challenges are to evaluate the reinjection effects that may occur and to design proper reinjection schemes for geothermal reservoirs.

Tracer testing is a standard method for tracing mass transport within a geothermal reservoir and can be a valuable tool in the design and management of production and injection operations (Horne, 1985; Niibori, 1995; Pruess, 2002). Shook (2001) discusses the potential application of tracer data to provide relatively simple reservoir properties and to make a prediction of thermal breakthrough. Such knowledge of the flow field offers a means of identifying problems with, and optimizing, injection. Through numerical simulation, one may further predict the onset of cooling in produced fluids. It may be advantageous if we can determine the reinjection conditions such as well location and injection flow rate, based on field tracer data obtained from a pair of exploratory wells.

The advection-dispersion equation (ADE) is widely used in hydrology to model transport of dissolved chemicals in subsurface water (Bear, 1972). Numerous field experiments for the solute transport in highly heterogeneous media demonstrate that solute concentration profiles exhibited anomalous non-Fickian growth rates, skewness, sharp leading edges and so-called ‘‘heavy tails’’ (Hatano and Hatano, 1998; Levy and Berkowitz, 2003; Benson et al., 2000). The ADE, however, cannot predict these effects.

One promising approach developed to model contaminant transport in heterogeneous media of fractal geometry is the fractional advection-dispersion equation (fADE) (Fomin et al, 2011). Through the use of fractional derivatives in time and space, the fADE can account for effects of solute transport retardation due to solute exchange between fracture network and rock matrix in fractured reservoirs. They suggest that calibrating the mathematical model and specifying the values of parameters in the mathematical model may provide important information about the geological structure of the rocks.

The aim of this study is to evaluate the applicability of the fADE model in the prediction of mass transport in complex fractured media. In this study, we utilize a 3D fracture network to produce numerical data of tracer responses. The fracture network model is based on a fractal fracture network model, and a number of disk-shaped fractures are generated assuming the fractal correlation based on the power-law relationship between the fracture length and the number of fractures. Through the synthetic tracer results, we are able to evaluate the effects of fractal geometry of fracture distributions on anomalous mass transport in a fractured reservoir. The fADE mathematical model is applied to fit the numerical tracer results simulated by the fracture network model and to predict the mass transport between an arbitrary set of wells, based on a tracer test result taken for a pair of wells with a certain interval.

## 2. fADE Model

### 2-(1) Governing Equation

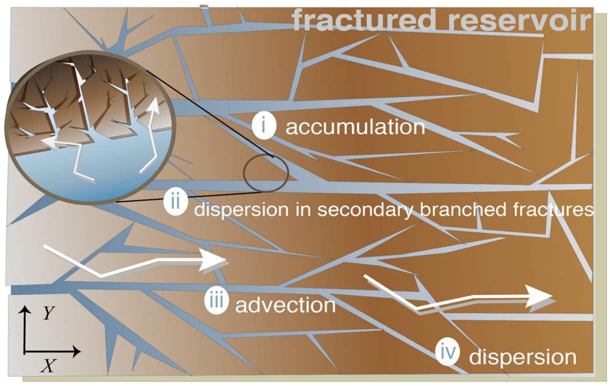
Fomin et al. (2011) assumed that a fractured porous reservoir is composed of two parts: rock matrix and fracture network based on fractal geometry. A schematic sketch of a fractured porous reservoir is presented in Fig. 1. The fracture network consists of a number of fractures whose length distribution follows the fractal geometry. The overall morphology can be viewed as a tree-like pattern. Primary tracers are mainly transported though the macroscopic stem of the tree-like fractures, and a certain portion of the tracers diffuse through the wall of the stem into the secondary fractures (mesoscale). Furthermore, the tracer into the rock matrix is transferred by diffusion from the macroscopic fractures. An equation of mass transport in the above fractured reservoir can be presented in the following form:

$$\frac{\partial C}{\partial T}+b\frac{\partial^{\gamma} C}{\partial T^{\gamma}}=-\frac{1}{Pe}\frac{\partial J}{\partial X}-\frac{\partial C}{\partial X} \tag{1}$$

where

$$J=\frac{1}{2}\left(\frac{\partial^{\alpha} C}{\partial X^{\alpha}}+\frac{\partial^{\alpha} C}{\partial(-X)^{\alpha}}\right) \tag{2}$$

Note that the Eqs. (1) and (2) have already been converted into non-dimensional form with the proper characteristic scales (Fomin et al., 2011). $C$, $T$, and $J$ are the concentration in a fractured reservoir, time, and the dispersive mass flux in a fractured continuum in the x-direction, respectively. $b$ is the factor of retardation processes and $Pe$ is the Peclet number, respectively. $\alpha$ ($0<\alpha<1$) and $\gamma$ ($0<\gamma<1$) are the order of fractional spatial and temporal derivatives, respectively. If the pore medium is composed of fractal structure or has a fractal distribution, the mass exchange from macroscopic fractures into matrix rocks can be expressed with fractional temporal derivatives. Namely, the effect can be expressed by the second term of left-hand side in Eq. (1). The mass flux in the reservoir, which is composed of fractal fracture network, can be defined as Eq. (2) with spatial fractional-derivative. While a term of describing diffusion into the surrounding rocks, which is assumed in the equation derived by Fomin et al. (2011), may be an important aspect of mass transport, it is deliberately neglected here to avoid possible interactions with the non-Fickian dispersion within the fractured reservoir.




$$\underbrace{\frac{\partial C}{\partial T}}_{\text{i}}+b\underbrace{\frac{\partial^{\gamma} C}{\partial T^{\gamma}}}_{\text{ii}}=-\underbrace{\frac{1}{Pe}\frac{\partial J}{\partial X}}_{\text{iii}}-\underbrace{\frac{\partial C}{\partial X}}_{\text{iv}}$$

$$J=\frac{1}{2}\left(\frac{\partial^{\alpha} C}{\partial X^{\alpha}}+\frac{\partial^{\alpha} C}{\partial(-X)^{\alpha}}\right)$$

**Figure 1.** Schematic of the fADE in a fractured aquifer.

### 2-(2) Finite Difference Solution of the fADE

Fomin et al. (2011) can only obtain an analytical solution of the fADE in the particular case when mass transport along the aquifer was dominated by the advection, and the first term in the right-hand side of the Eq. (1) was ignored. In order to make the practical application of fADE available, we present a finite difference approach to solve the Eqs. (1) and (2).

We will assume that $C(X, T)$>=0 over the region 0<= $X$ <= $L$ , 0<= $T$ <= $T_{max}$ . Define $t_n = n\Delta t$ to be the integration time 0<= $t_n$ <= $T_{max}$, $\Delta x$ >0 to be a grid size in spatial dimension where $\Delta x = L / NX$, $x_i = i\Delta x$ for $i$ =0,..., $NX$ so that 0<= $x_i$ <= $L$. Let an approximation to $C(x_i, t_n)$. We have the finite difference solution combining Eqs. (1) and (2), which is discretized in time using an implicit (Euler) method:

$$\begin{aligned}
&C_i^{n+1}+\frac{\Delta T}{2\Delta X}C_{i+1}^{n+1}-\frac{\Delta T}{2\Delta X}C_{i-1}^{n+1}-\frac{1}{2Pe\Gamma(2-\alpha)}\\
&\left[\sum_{k=0}^{i}(C_{k+1}^{n+1}-C_k^{n+1})w_k^i-\sum_{k=0}^{i-1}(C_{k+1}^{n+1}-C_k^{n+1})w_k^{i-1}\right.\\
&\left.+\sum_{k=i}^{NX}(C_{k+1}^{n+1}-C_k^{n+1})w_i^k-\sum_{k=i-1}^{NX}(C_{k+1}^{n+1}-C_k^{n+1})w_{i-1}^k\right]\\
&=C_i^n-b\Delta T^{1-\gamma}\sum_{k=0}^{n}w_i^{\gamma}C_i^{n+k-1}
\end{aligned} \tag{3}$$

In this work, we use the prescribed-flux boundary, which has a prescribed flux at the inlet $x = 0$ and a free drainage at the outlet $x = L$ (Zhang et al., 2007).

## 3. Numerical Analysis

### 3-(1) Flow Analysis by FRACSIM-3D

A numerical model FRACSIM-3D, which is developed by Watanabe and Takahashi (1995) and Jing et al. (2000), is proved to be an appropriate approximate model capable to address simultaneously the problems associated with hydraulic stimulation, fluid circulation and heat extraction. The FRACSIM-3D code has been used to model the Hijiori and Soultz reservoirs. In the Hijiori model, the simulation tracer responses have been in reasonable agreements with the observed results (Jing et al., 2000).

In FRACSIM-3D fractures are generated stochastically within a fracture generation area as shown in Fig. 2. The fracture centers are uniformly random, with the length distribution fractal and orientation controlled by specifying ranges of azimuths and frequencies for a number of fracture sets. The fracture realization stops when the fracture density (fractures per meter) reaches a specified level. The quantity of fluid flow from block to block is controlled by Darcy's law with the permeability distributions from each fracture governed by the sum of the products of the cubes of the fracture apertures and the fracture intersection. Accounting for the mass conservation equation, fluid flow is assumed to be laminar and controlled by the continuity equation:

$$\frac{\partial}{\partial x}\left(K_x\frac{\partial}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_y\frac{\partial P}{\partial y}\right)+\frac{\partial}{\partial z}\left(K_z\frac{\partial P}{\partial z}\right)=0 \qquad (4)$$

where *Ki* is the permeability for each surface of a grid. *P* is the pressure. A natural fluid flow is assumed to take place from the injection point toward the production point and is simulated by pressure gradient between inlet boundary and outlet boundary side.

A method for tracking tracer particles has been built in the model (Jing et al., 2000). The tracer analyses are based on the premise that tracer substances are particle ensembles and each tracer particle travels from the injection well to the production well located in the above flow. The transit time is calculated and added to the total time elapsed since injection for this particle and the block coordinates updated to those of the new block.

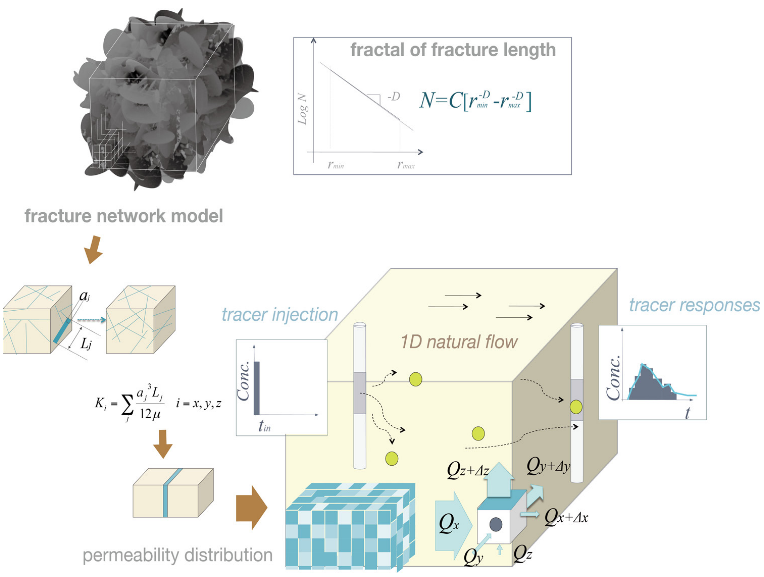



**Figure 2.** Schematic of tracer response analysis based on the fractured reservoir model.

### 3-(2) Application of the fADE to the Numerical Tracer Responses

The Eqs. (1) and (2) were converted into non-dimensional form with the proper characteristic scales. The tracer results obtained by FRACSIM-3D also need to be rewritten in non-dimensional forms for comparison with the fADE solutions. The proper characteristic scales are defined as follows. The characteristic scale for the variable x along the reservoir is the well spacing. The characteristic scale for time represents average tracer travel time. The initial concentration of tracer from injection well, which includes the recovery rate and the mixing ratio of tracer responses, can be used as the characteristic scale for solute concentration.

We used calculated data on 50 tracer responses from different fracture network models in which the random seeds varied. First of all, the length of well spacing was set to 50 m. The fADE mathematical model was applied to fit each numerical tracer results simulated by FRACSIM-3D. For comparison, the ADE also used to characterize each tracer responses in addition to the fADE. Note that ADE is the special case of fADE in which the value of $\alpha$ and $\gamma$ is set to 1. Namely, the ADE is not composed of fractional derivatives but includes the retardation term as described in the second term in left-hand side of Eq. (1). For each optimal value of $\alpha$, $\gamma$, $b$, and $Pe$ were estimated to minimize the root- mean-squared error (*RMSE*):

$$RMSE=\sqrt{\frac{1}{N}\sum_i^N\left(\frac{C_{ie}-C_{ic}}{C_{ic}}\right)^2} \qquad (5)$$

where $C_{ie}$ is the estimated concentration, $C_{ic}$ is the calculated concentration, and $N$ is the number of observed concentration data at a particular observation point.

Consequently, the determined constitutive parameters were then used in the fADE and ADE models to predict the tracer responses in the case where the well spacing was extended to 80 m and each of the tracer responses predicted by the mathematical models were compared with those calculated using FRACSIM-3D for the well interval of 80 m.

To compare the two models considered, both the determination coefficient ($r^2$) and *RMSE* were used as two criteria to reflect the goodness of simulation. $r^2$ can be expressed as:

$$r^2=1-\frac{\sum_i^N\left(C_{ie}-C_{ic}\right)^2}{\sum_i^N\left(C_{ic}-\bar{C}_{ic}\right)^2} \qquad (6)$$

where $C_{ic}$ represents the mean values of $C_{ic}$. Finally, we calculated the mean value of $r^2$ and RMSE for each 50 tracer responses.

## 4. Results and Discussion

The FRACSIM-3D produced numerical data of tracer responses with different fracture densities. Problem parameters are given in Table 1. Here we present re-

**Table 1.** Parameters used for the simulation.

| Parameters | | Value |
|---|---|---|
| Calculation domain | [m] | 100x100x70 |
| Element size | [m] | 1x1x1 |
| Fractal dimension | | 2.5 |
| Fracture radius | r [m] | 1~25 |
| Fracture aperture | [m] | $1.0\times10^{-4}\times r$ |
| Well length | [m] | 50 |
| Injection pressure | [Pa] | $1.1\times10^{5}$ |
| Production pressure | [Pa] | $1.0\times10^{5}$ |
| Matrix permeability | [m$^2$] | 0 |
| Viscosity of water | [Pa s] | $1.826\times10^{-4}$ |
| Number of tracer particles | | $1.0\times10^{4}$ |

sults of three tests that were performed to evaluate the effects of fracture densities on tracer responses. The tracer responses at fracture densities of 19, 1.9, and 0.6 [1/m], respectively, are shown in Fig. 3. In order to compare the tracer response curves with Fickian behaviors, the ADE was applied to fit them, and the solutions are also plotted in Fig.3. The simulated result at fracture density of 19 [1/m] shows a relatively regular and symmetric shape and agreement with the solution of ADE in Fig. 3 (a). The result indicates that the transport has translated to Fickian. In contrast, the tracer response at fracture densities of 0.6 [1/m] is more irregular and in disagreement with the solutions of ADE, as shown in Fig. 3 (c). The feature of tracer responses can be regarded as non-Fickian.

The horizontal cross-sections of permeability distribution for each block surface, in the three cases when fracture density varies, are presented in Fig. 4. The distribution of permeability is governed by the sum of product from each penetrating fracture. Thus, we can evaluate the effects of fracture distributions on fluid and mass transport using these distributions of permeability. In addition, we are able to visualize a 3D map of some particle pathways from injection well to production well at fracture density of 19, 1.9, and 0.6[1/m], respectively (see Fig. 5). The permeability distribution at high fracture density seemed to be heterogeneous by the existence of a number of large fractures. However, the flow paths of tracers from injection well to production well is along x direction at fracture density of 19[1/m] as shown in Fig. 5 (a). This result indicates that fractures are much enough to occupy within whole the calculation area, and the reservoir is assumed to become a relatively homogeneous fluid medium. Thus, it is assumed that fracture network does not interrupt the movement of tracer particles.

On the other hand, a sparse permeability distribution is shown in Fig. 4 (c), whose fracture density is set to 0.6 [1/m]. We can clearly distinguish the fracture network from rock matrix. Then, the tracer particle paths do not move in a straight line along x-axis as shown in Fig. 5 (c). That gives evidence for the existence of fast preferential flow paths, which are attributable to heterogeneous arrangement of fractures. Since individual tracer particles move along various flow paths and cause a broad range of travel time distribution, tracer response curve seems to be identified as non-Fickian behavior.

An arbitrary set of the calculated and fitted tracer response at distances of 50 m at fracture density of 1.9 [1/m] is shown as Fig.6 (a). The 50 tracer responses for different fracture network were individu-

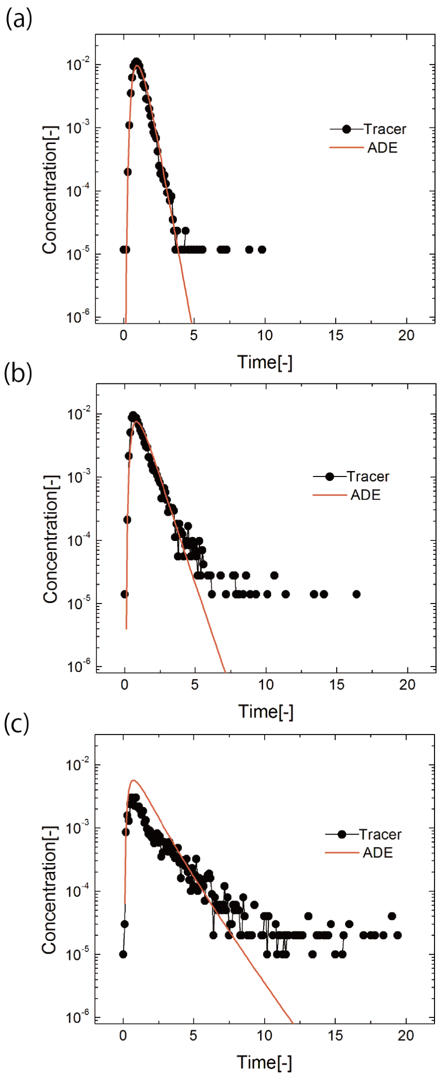


**Figure 3.** Profile of tracer responses at fracture density of (a) 19 [1/m], (b) 1.9[1/m], and (c) 0.6[1/m].

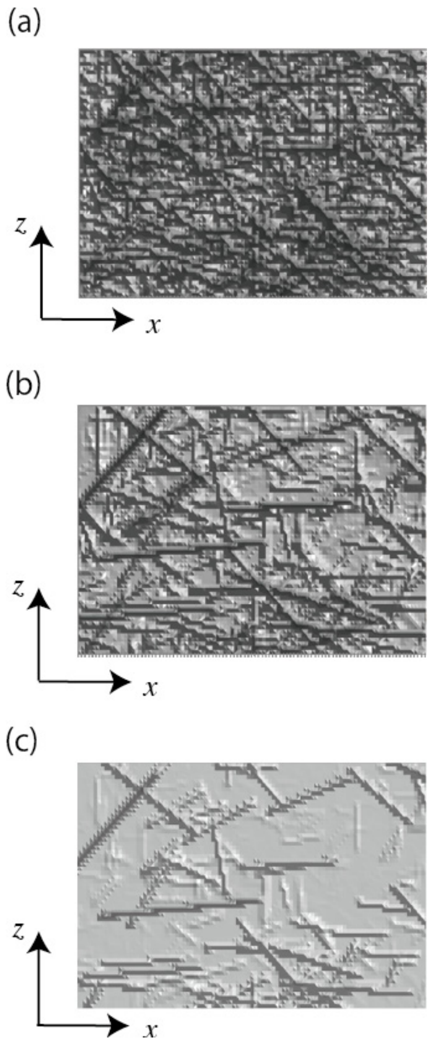


**Figure 4.** Horizontal cross-sections of permeability distribution at fracture density of (a) 19 [1/m], (b) 1.9[1/m], and (c) 0.6[1/m].

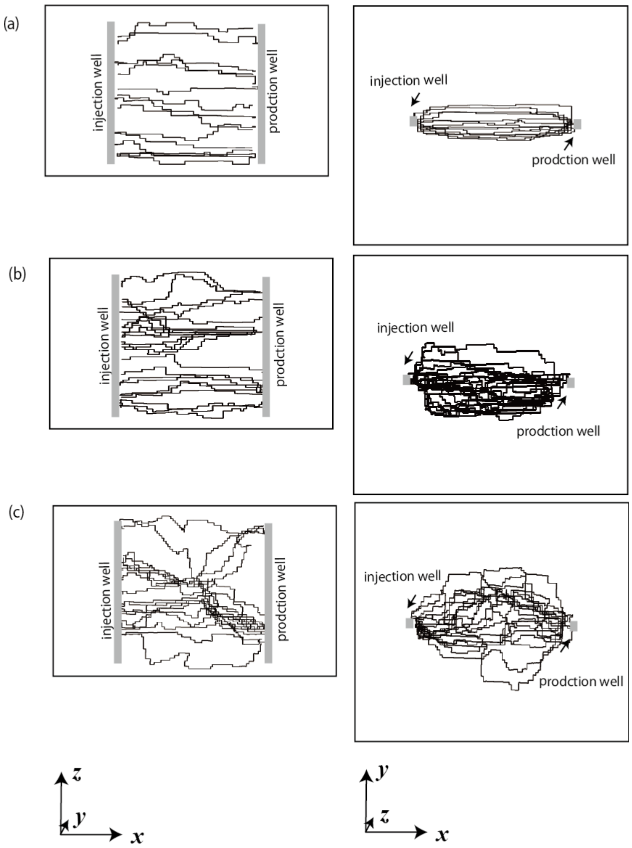



**Figure 5.** Particle travel paths at fracture density of (a) 19 [1/m], (b) 1.9[1/m], and (c) 0.6[1/m].

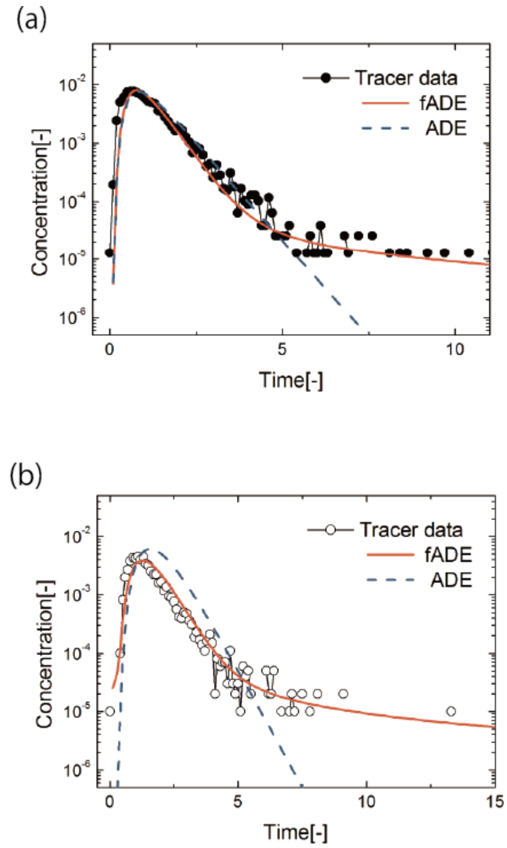



**Figure 6.** (a) Comparison of the temporal concentration profiles estimated by the fADE (solid line), the ADE (dashed line), and the calculated data at distance of 50m from FRACSIM-3D (symbols) (b) predicted by the fADE (solid line), the ADE (dashed line) and calculated (symbols) temporal concentration profiles at distance of 80m.

ally fitted with the fADE and the ADE. The concentration data in the figure have already been normalized by the suitable values before the parameters in two models were estimated. The mean values of estimated parameters for each tracer responses are listed in Table 2 for the fADE and the ADE. There are profound discrepancies between the calculated tracer result at 50 m and its fitted by the ADE. The fitting results of the ADE at the tails of tracer responses are slightly smaller than the calculated results. Compared to the ADE, the fADE provides better simulation results at the tailing parts of tracer behaviors, as evident from Fig. 6 (a). This indicates that the fADE is capable of describing the long tail of tracer responses somewhat.

Consequently, the estimated constitutive parameters in the fADE and ADE models were then used to predict the tracer responses in the case where the well spacing was extended to 80m and the tracer responses predicted by the mathematical models were compared with those calculated using FRACSIM-3D for the well interval of 80m. The simulated and predicted tracer concentrations at distance of 80 m are shown in Fig. 6 (b). The associated $r^2$ and *RMSE* values are listed in Table 2. ADE is less satisfactory to predict the tracer responses, as illustrated in Fig. 6 (b). Compared to the ADE, the predicting result of the fADE is more satisfactory. This is also indicated by the larger $r^2$ and small RMSE values of the fADE than those of the ADE, as shown in Table 2. The fADE captures the evolution of tracer responses, especially their tails. This result demonstrates the applicability of the fADE in characterizing anomalous transport in the fractured reservoir. As shown in Table 2, there were no significant differences in the parameter values of $\alpha$, $Pe$, $b$ of the fADE and ADE models. In contrast, the value of $\gamma$ varied between the fADE and ADE. Since the fADE including the time fractional derivative is able to describe the effects of dispersion process into secondary fractures, it seems suitable to interpret the anomalous tracer behaviors in fractal fracture networks.

The present study, which was based on the comparison of the mathematical and numerical models, shows that the fADE model provides a useful tool for characterizing the anomalous mass

**Table 2.** Average of best-fit parameters, determination coefficient $r^2$ and root mean square error RMSE for fADE and ADE model obtained by estimation of 50 tracer responses. The values of α and γ in ADE are given as 1.

| | Average Best-Fit Parameters | | | | Average Estimation | |
|---|---|---|---|---|---|---|
| | $\gamma$ | $\alpha$ | $b$ | $Pe$ | $r^2$ | RSME |
| fADE | 0.2 | 0.98 | 0.1 | 7.45 | 0.835 | 1.996 |
| ADE | 1* | 1 | 0.11 | 7.04 | 0.749 | 3.475 |

transport behavior in complex fractured reservoirs. Furthermore, the fADE possesses a potential to predict the mass transport based on tracer response curves. According to Shook (2001), velocity and travel time of the thermal front is possible to be estimated by analyzing a tracer response curve. If we develop a tracer analysis method to predict thermal breakthrough based on the fADE, the method may be advantageous to design injection conditions, such as well locations, flow rates, or temperatures. The dispersion of injected water into secondary fractures, which is taken account in the fADE, can affect the thermal breakthrough behavior and should be considered in the prediction of reservoir behaviors. Future work is expected to expand the fADE model with respect not only to mass transfer but also to heat transfer model.

## 5. Summary

The fractional Advection Dispersion Equation (fADE) can be a useful tool for analyzing tracer transport behaviors in fractured reservoirs. Through the use of fractional derivatives in time and space, the fADE can account for effects of solute transport retardation due to solute exchange between fracture network and rock matrix in fractured reservoirs. In this paper we presented a finite difference approach to solve the equations in fADE.

A 3D simulation code for flow analysis (FRACSIM-3D) was utilized to produce numerical data of tracer responses. FRACSIM-3D was based on a fractal fracture network model, and a number of disk-shaped fractures are generated assuming the fractal correlation based on the power-law relationship between the fracture length and the number of fractures. The density of the natural fractures was varied in the numerical analyses. It has been shown that the FRACSIM-3D reproduced highly anomalous behaviors with heavy tails at low fracture densities. The feature of anomalous tracer responses was often observed in field or laboratory tests. The tracer responses that were not adequately explained by the advection - dispersion equation (ADE) were therefore regarded as non-Fickian transport. The anomalous tracer responses produced by FRACSIM-3D was attributable to the preferential pathways that arose because of the degree of fracture connectivity.

The fADE mathematical model was applied to fit the numerical tracer results characterized by non-Fickian. For comparison, the ADE was also used to describe the tracer responses in addition to the fADE. The curve fitting enabled us to determine the constitutive parameters in both the mathematical models by analyzing the tracer data obtained for a well interval of 50m. The determined constitutive parameters were then used in the fADE and the ADE models to predict the tracer responses in the case where the well spacing was extended to 80m. It was demonstrated that the tracer responses predicted by the fADE model was in reasonable agreement with the numerically obtained data by FRACSIM-3D, while the ADE model produced tracer curves which deviated significantly from the FRACSIM-3D results particularly for long-term behaviors.

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