



Fractional diffusion modeling of heat transfer in porous and fractured media



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ABSTRACT

Fracture–matrix interactions strongly affect anomalous heat transfer in geological sites. This study investigates effects of the interactions between fractures and rock matrix by using the method of *multiple interacting continua* (MINC). The MINC generates different temperature histories for varied fracture spacings. Two analytical solutions of each porous model and fracture model are used to fit the numerical results for temperature histories due to cold-water injection. The porous model is in good agreement with the result for small fracture spacing, while a solution of the fracture model fits the result for large fracture spacing. The MINC yields intermediate behaviors in between a porous medium and a single fracture. A fractional heat transfer equation (fHTE) has been developed to describe anomalous thermal diffusion in a fractured reservoir. The fHTE accounts for heat flux from fracture into matrix by using a temporal fractional derivative. The fHTE can capture numerical results for temperature histories with different fracture spacings. The fracture spacing has correlations to the fHTE best-fit parameters (i.e., the orders of fractional derivatives and the retardation parameters). The fHTE with varying time fractional derivatives can cover descriptions of subdiffusion, Fickian diffusion, and superdiffusion.

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1. Introduction

Natural fracture networks significantly control hydrodynamic and thermodynamic behaviors in geological fields (e.g. geothermal energy, deep geologic storage of nuclear waste, induced saltwater intrusion, hydraulic fracturing, and carbon dioxide sequestration). Fast flow paths through fractures may lead to rapid migration, while interactions of flow with the rock matrix (i.e., advective imbibition, diffusion, or adsorption) affect the retardation of fluid and heat.

The fracture–matrix interactions are treated with dual-continuum approaches, which include the classical double-porosity model [1,42], the dual-permeability concept [9], and the more rigorous dual-continuum generalization of the method of *multiple interacting continua* (MINC) [19,25]. In the double-porosity concept, a network of interconnected fractures forms

the flow paths, and the embedded rock matrix is the subdomains exchanging mass and heat between the flow domain and the stagnant domains. This concept assumes that approximate thermodynamic equilibrium locally exists between fracture and matrix, that is, fracture–matrix exchanges occur instantaneously [8]. In contrast, the MINC method is able to describe gradients of pressures, temperatures, or concentrations inside the matrix by subdividing individual matrix blocks. Although improved capacities of computer simulations allow us to use very large amount of grids and huge computer storage spaces, the main difficulty is to determine numerous site-specific input parameters. Unfortunately, most measurement data are obtained from limited samples and do not accurately describe a fractured medium. Inverse problem analyses (e.g., iTOUGH2 [13,12] and stochastic approaches [41,26]) have been improved but tend to be computationally intensive.

For idealized and simplified systems and conditions, it is possible to solve mathematical models by analytical techniques, which would be attractive to characterize reservoir properties during early phases of developments. Classical modeling for heat transfer in a single fracture treats heat exchange between a fracture and the

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Nomenclature

a	arbitrary constant	λ	thermal conductivity
a'	thermal diffusivity	λ'	effective thermal conductivity
b	fracture aperture	Φ	memory function
C_p	heat capacity	ρ	density
d	arbitrary constant	θ	index of permeability reduction
J	heat flux		
K	permeability		
k	coefficient including physical properties and the structure of porous rocks	<i>Subscript</i>	
t	time	0	initial
T	temperature	1	reservoir
u	fluid velocity in the fracture	c	constant
x	horizontal coordinate	f	fluid
X	non-dimensional distance	in	injection
z	vertical coordinate	m	matrix
β	order of temporal fractional derivative	r	rock
γ	order of temporal fractional derivative	s	surrounding rock
κ	coefficient depending on the shape of porous blocks	w	water

embedded rock matrix. The purpose of this study is to develop a heat transfer equation based on the fractional calculus. Recently, fractional calculus has been applied to modeling methodologies and have attracted interest in several fields, for instance, fluid mechanics [20], rheology [4] bioengineering [24], and hydrological modeling [27,44,2]. Temporal fractional derivatives can be used to describe diffusion into matrix and/or into surrounding rocks where fractures show self-organized fractal distributions [15,16,32,33]. The advantage of using fractional calculus is its ability to characterize phenomena in heterogeneous media with few parameters. The drawback is that the physical meaning of the constitutive parameters is still unknown.

Fractional differential equations for heat transfer has been studied [28,23,10,17]. Little is known about the relationship between the order of time fractional derivatives and geological structures. First, we show conventional modeling approaches for heat transfer and compare with the fractional heat transfer model. Numerical simulation results are obtained from the MINC method to reveal insights into the physical meaning of fractional derivatives in heat transfer.

2. Methodology development

2.1. Conventional mathematical heat transfer models

Bodvarsson [5] derived the basic equation for subsurface temperature fields in a homogeneous porous medium with intergranular flow. The governing equation can be written as:

$$\frac{\partial T_1}{\partial t} = -\frac{\phi_w \rho_w C_{pw}}{\rho C_p} u \frac{\partial T_1}{\partial x}, \quad (1)$$

where $\overline{\rho C_p} = \phi_w \rho_w C_{pw} + (1 - \phi_w) \rho_r C_{pr}$. T_1 is temperature of the flow domain, t is time, and x is distance. ρ_w and ρ_r are the density of water and rock, respectively. C_{pw} and C_{pr} are the heat capacities of water and rock. ϕ_w is the porosity, and u is the fluid velocity. This equation assumes that uncompressed fluid flows in a homogeneous porous medium. The rock grains are so small that there is a perfect temperature contact between the fluid and the rock grains. Because convection is dominant in most geothermal hydrothermal systems [43], thermal conduction was neglected. This model will be referred to as the *porous model* in this paper. The term on the left hand side

describes heat accumulation in the porous medium. The term on the right hand side represents convection.

Lauwerier [21] developed an analytical solution for heat transfer with heat loss into confining beds according to the Fourier law. Heat exchange between a single flowing region (fracture) and stagnant regions is considered. Bodvarsson and Tsang [6] presented a differential equation for a single fracture surrounded by confining rock masses as follows:

$$\begin{aligned} \frac{\partial T_1}{\partial t} &= -\frac{\phi_w \rho_w C_{pw}}{\rho C_p} u \frac{\partial T_1}{\partial x} + \frac{\lambda_r}{b \rho C_p} \frac{\partial T_s}{\partial z} \Big|_{z=0}, \\ \rho_r C_{pr} \frac{\partial T_s}{\partial t} &= \lambda_r \frac{\partial^2 T_s}{\partial z^2}; \end{aligned} \quad (2)$$

where T_1 is the temperature of the fluid in the fracture and T_s is the temperature of the surrounding rock masses. λ_r is the thermal conductivity of the rock and b is the fracture aperture. z is the distance from the fracture, which is perpendicular to the x -axis. The term on the left hand side accounts for heat accumulation in the fracture. The first and second terms on the right hand represent convection in the fracture and heat loss into the confining rocks, respectively. Thermal equilibrium is assumed to take place instantaneously between water and rocks, so that anywhere in the fracture rocks have the same temperature as the surrounding fluid. The heat flux into the surrounding rocks is given by

$$J_s = -\lambda_r \frac{\partial T_s}{\partial z} \Big|_{z=0}. \quad (3)$$

This model can express thermal diffusion from a single fracture into the surrounding rocks following the Fourier law. We call this mathematical model the *single-fracture model* in this paper.

2.2. The time fractional diffusion model

Fractional diffusion equation has been used to describe anomalous diffusion processes, which do not follow the Fick's law and are called *non-Fickian* solute transport [27,44]. Fick's law of solute diffusion and Fourier's law of heat conduction both are empirical laws. Fick's law describes that mass flux is proportional to the concentration gradients, while Fourier's law describes that heat flux is proportional to temperature gradients, respectively. A fractional advection–dispersion equation (FADE) can model mass transport

in a heterogeneous medium, with a non-integer-order derivative (fractional derivative) on the time and/or space terms of the equations [36,37]. The time fADE includes memory effects, allowing solute particles to reside in the system for long periods. While traditional mobile-immobile models assume that particles move between the mobile and immobile phases (i.e., flow and stagnant regions) at an instantaneous rate proportional to the difference in concentration, the time fADE governs long-term release from the immobile phase by using a memory function according to a power law [18].

Let us consider heat transfer process in a similar way to solute transport. Supplementing the single-fracture model (2) by the following the boundary and the initial conditions:

$$\begin{aligned} t = 0, \quad T_1 &= T_0; \\ z = 0, \quad T_s &= T_1(t, x); \\ z \rightarrow \infty, \quad T_s &\rightarrow T_0. \end{aligned} \tag{4}$$

where T_0 is the initial temperature. It becomes clear that the temperature of the surrounding rock T_s can be expressed through the temperature of flow domain T_1 . As it follows from the monograph of Fomin and Chugunov [14], by applying the Duhamel theorem, the temperature of the surrounding rocks can be presented as follows:

$$T_s(z, t) = \int_0^t \frac{\partial T_1}{\partial \xi} T_a(z, t - \xi) d\xi. \tag{5}$$

In Eq. (5), $T_a(z, t)$ is the auxiliary function that is defined by the single-fracture model (2) and the boundary conditions (Eq. (4)). If the boundary condition at $z = 0$ is replaced by the following one: $z = 0, T_a(z, t) = 1$, the function $T_a(z, t)$ can be readily obtained in the following form:

$$T_a(z, t) = \operatorname{erfc}\left(\frac{z}{2\sqrt{a_r t}}\right), \tag{6}$$

where $a_r = \lambda_r / (\rho_r C_{pr})$ known as thermal diffusivity. It characterizes the velocity of propagation of isothermal surfaces in a body. Accounting for Eqs. (5) and (6), the heat flux (Eq. (3)) can be rewritten as follows:

$$J_s = - \int_0^t \frac{\partial T_1}{\partial \xi} \Phi(t - \xi) d\xi, \tag{7}$$

where Φ can be referred as a memory function given by

$$\Phi(t) = -\lambda_r \frac{\partial T_a}{\partial z} \Big|_{z=0}. \tag{8}$$

If the temperature T_a is given by Eq. (6), then

$$\Phi(t) = \frac{\lambda_r t^{-1/2}}{\sqrt{\pi a_r}}. \tag{9}$$

Substituting Eq. (9), the heat flux (Eq. (7)) can be rewritten in the term of Caputo fractional derivative [34] as follows:

$$J_s = - \frac{\lambda_r}{\sqrt{a_r}} \frac{\partial^{1/2} T_1}{\partial t^{1/2}}. \tag{10}$$

Accounting for this expression (Eq. (10)), the single-fracture model (2) completely determines the temperature in the fracture T_1 :

$$\frac{\partial T_1}{\partial t} + \frac{\phi_w \rho_w C_{pw}}{\rho C_p} u \frac{\partial T_1}{\partial x} + d_s \frac{\partial^{1/2} T_1}{\partial t^{1/2}} = 0, \tag{11}$$

where $d_s = \frac{\lambda_r}{b \rho C_p \sqrt{a_r}}$. Thus, even in the widely known traditional model of heat transfer in a single fracture, the governing equation contains the fractional derivative ($=1/2$).

Fomin and Chugunov [14] have shown the wide range of the mass transfer processes in complex media (such as fractured reservoirs and surrounding porous rocks). Eqs. (9) and (10) for the diffusive transport preserve their form and in the general case can be presented as:

$$\Phi(t) = \frac{\kappa_s t^{-\beta}}{\Gamma(\beta)}, \tag{12}$$

$$J_s = -\kappa_s \frac{\partial^\beta T_1}{\partial t^\beta}; \tag{13}$$

where $\Gamma(\beta)$ is a Gamma function, κ_s is the coefficient that depends on the physical properties and the structure of the porous rocks and β is the order of the fractional derivative ($0 < \beta \leq 1$). Accounting for the similarity of the processes of diffusion and heat transfer it would be quite natural to assume that Eqs. (12) and (13) are applicable also for modeling the processes of heat transfer.

2.3. Thermal diffusion in geothermal reservoirs

Faults govern fluid flow, which consist of impermeable fault cores and permeable damage zones [7]. Many quantitative studies on fault zones have shown that fracture density at damage zones decreases with increasing distance from the fault core [11]. The main conduit is assumed to be near a fault core on a damage zone. Let the fault core spread horizontally along the x -axis with the main conduit is along the fault core and with one-dimensional flow in the x direction. Savage and Brodsky [35] suggested that fracture density at a damage zone decays according to a power law. Suzuki et al. [40] suggested that a power-law model of fracture density is applicable to the fault zone architectures in a geothermal reservoir. The heat flux from a main conduit into the surrounding damage zone will be given by Eq. (13) [40], where

$$\beta = (1 + \theta) / (2 + \theta), \tag{14}$$

$$k_s = k_c(\theta) \lambda'_r (a'_r)^{-\beta}; \tag{15}$$

where k_c is the coefficient that depends on the structural peculiarities of the rocks. θ is the index of the diffusivity reduction. λ' and a'_r are the effective conductivity and thermal diffusivity of the surrounding rocks.

In addition, we assume that a reservoir is dissected by fractures into a set of multiple porous blocks, which are not in thermodynamic equilibrium with fluid conducting fractures. Then, two temperatures should be considered: the temperature of the conducting fluid channels (i.e., main fractures), T_1 , and the temperature of the porous matrix, T_m . The heat flux from the matrix can be defined by the formula similar to Eq. (13):

$$J_m = (1 - \phi_c) \kappa_m \frac{\partial^\gamma T_1}{\partial t^\gamma}, \tag{16}$$

where γ is the order of the fractional derivative ($0 < \gamma \leq 1$). $\kappa_m = k_f \lambda'_m (a'_m)^{-\gamma}$, k_f is the coefficient that depends on the shape of the porous blocks and their structure, λ'_m and a'_m are the effective heat conductivity and diffusivity of the blocks, respectively, ϕ_c is the fraction volume of the fractures within the reservoir.

The heat balance equation for the mean across the reservoir temperature, T_1 , can be presented in the following form:

$$b \phi_c \rho C_p \frac{\partial T_1}{\partial t} + b \phi_w \phi_c \rho_w C_{pw} u \frac{\partial T_1}{\partial x} = \phi_c J_s + J_m. \tag{17}$$

Accounting for the heat fluxes into surrounding rocks (Eq. (13)) and into matrix (Eq. (16)), Eq. (17) can be converted to the following form:

$$\frac{\partial T_1}{\partial t} + \frac{\phi_w \rho_w C_{pw}}{\rho C_p} u \frac{\partial T_1}{\partial x} = - \frac{\kappa_s}{b \rho C_p} \frac{\partial^\beta T_1}{\partial t^\beta} - \frac{\kappa_m (1 - \phi_c)}{b \phi_c \rho C_p} \frac{\partial^\gamma T_1}{\partial t^\gamma}, \quad (18)$$

or denoting $d_s = \frac{\kappa_s}{b \rho C_p}$ and $d_m = \frac{\kappa_m (1 - \phi_c)}{b \phi_c \rho C_p}$,

$$\frac{\partial T_1}{\partial t} + \frac{\phi_w \rho_w C_{pw}}{\rho C_p} u \frac{\partial T_1}{\partial x} + d_s \frac{\partial^\beta T_1}{\partial t^\beta} + d_m \frac{\partial^\gamma T_1}{\partial t^\gamma} = 0. \quad (19)$$

From Eq. (19) it follows that if the reservoir is thermally insulated and there is no blocks, then $\kappa_s = \kappa_m = 0$ and $\phi_c = 1$. As a result, Eq. (19) reduces to the porous model (1). If $\kappa_m = 0$ and $\phi_c = 1$ and $\beta = 1/2$, then Eq. (19) reduces to the single-fracture model (2).

The initial and boundary condition for Eq. (17) can be taken as follows:

$$\begin{aligned} t = 0, \quad T_1 = T_0 = \text{const.}; \\ x = 0, \quad T_1 = T_{inj}; \end{aligned} \quad (20)$$

where T_0 is the initial reservoir temperature. T_{inj} is the temperature at the reservoir at its inlet.

The non-dimensional variables can be introduced as:

$$\begin{aligned} X = \frac{x}{l}; \quad \tau = \frac{t}{t_0}; \quad t_0 = \frac{l}{u}; \quad T = \frac{T_1 - T_0}{T_{inj} - T_0}; \\ e_s = d_s t_0^{1-\beta}; \quad e_m = d_m t_0^{1-\gamma}; \quad v = \frac{\phi_w \rho_w C_{pw}}{\rho C_p}; \end{aligned} \quad (21)$$

where l is the well spacing. t_0 is the characteristic time given by fluid velocity u and the well spacing l . e_s and e_m can be referred the retardation parameters. In the non-dimensional variables, the governing equation in the reservoir (Eq. (19)) and boundary conditions (Eq. (20)) can be rewritten as

$$\frac{\partial T}{\partial \tau} + v \frac{\partial T}{\partial X} + e_s \frac{\partial^\beta T}{\partial \tau^\beta} + e_m \frac{\partial^\gamma T}{\partial \tau^\gamma} = 0. \quad (22)$$

$$\begin{aligned} \tau = 0, \quad T = 0; \\ X = 0, \quad T = 1; \end{aligned} \quad (23)$$

The initial boundary value problem admits an analytic solution [16], which has the following form

$$T(\tau, X) = \chi\left(\tau - \frac{X}{v}\right) \psi\left(\tau - \frac{X}{v}, X\right), \quad (24)$$

where χ is the Heaviside function,

$$\begin{aligned} \psi(\tau, X) = 1 - \frac{1}{\pi} \int_0^\infty \exp\left[-\xi \tau - \frac{X}{v} (e_m \xi^\gamma \cos(\pi\gamma) + e_s \xi^\beta \cos(\pi\beta))\right] \\ \times \sin(\hat{\psi}(\xi, X)) \frac{d\xi}{\xi} \end{aligned} \quad (25)$$

where $\hat{\psi}(\xi, X) = \frac{X}{v} (e_m \xi^\gamma \cos(\pi\gamma) + e_s \xi^\beta \cos(\pi\beta))$.

3. Results

3.1. Effects of heat diffusion into matrix

In geothermal development, reinjection is one of most challenging problems for sustainable utilization of geothermal resources. Water is injected into a reservoir to maintain the pressure and the mass of fluid in place. Because water colder than the reservoir temperature might cool down the reservoir, we need to inject the water carefully. Let us examine the water injection problem. Suzuki et al. [40] investigated the effect of heat flux into surrounding rocks on the temperature breakthrough in a geothermal field. This study discusses the effects of thermal diffusion into rock

matrix. The general-purpose reservoir simulator TOUGH2 [30] was used to simulate fluid, mass, and heat flow.

The MINC in TOUGH2 can describe interactions between fractures and matrix [29,25]. In this study, ten sub-continua for each grid were implemented in the MINC partitioning process. The fracture continuum had a volume fraction of 0.045 and an *intrinsic* porosity of 0.5. Volume fractions of 0.051, 0.058, 0.067, 0.079, 0.097, 0.125, 0.176, and 0.3 were used for the other nine matrix continua with porosity of 0.1. The MINC grid consisted of 880 (= 80 × 11) blocks. The initial reservoir temperature was set to 200 °C, and the cold water at 100 °C was injected from the inlet with constant flow rate. Temperature was observed at the outlet under a constant production pressure. Gravity and capillary forces were neglected. The numerical properties are summarized in Table 1.

Fracture spacing, which is one of geometric parameters in the MINC, has been used to assess effects on reservoir performance (e.g., [31]). The simulated temperature profiles for different fracture spacings are shown in Fig. 1. The temperature for fracture spacing of 0.5 m declined rapidly. Because the volume of rock matrix was small enough, water migrating through the fracture cooled down inside the matrix instantaneously. This model coincided with a porous medium virtually, in which thermodynamic equilibrium locally exists between fracture and matrix. In contrast, the results for fracture spacing of 20 m yielded an early start of thermal breakthrough and a slow decline of the temperature. This result suggests that the fracture formed fast preferential flow paths and heat exchange between fracture and matrix occurred gradually. Our findings are consistent with Pruess and Wu [31].

3.2. Application of the time fADE

The temperature histories obtained by the MINC (Fig. 1) were compared with the solutions of the fHTE (Eq. (24)). The rock and water properties in the equations (i.e., fluid velocity, porosity, density, and heat capacity) were set to the same values as in the TOUGH2 simulation. The retardation parameter e_m and the order of fractional derivative γ were determined by an optimization approach to minimize the root mean squared error (RMSE) to fit the temperature history.

Fig. 2 shows the best-fit curves of the fHTE onto the simulated temperature histories of the MINC. The porous model (1) and the single-fracture model (2) were also used to fit the MINC results. The fitting errors (RMSE) and the coefficients of determination

Table 1
Numerical properties used in the MINC.

Property	
<i>Calculation domain</i>	
Thickness	68.57 m
Length	500 m
<i>Permeability</i>	
Fracture	$1.0 \times 10^3 \text{ m}^2$
Matrix	$1.0 \times 10^{18} \text{ m}^2$
<i>Porosity</i>	
Fracture	0.5
Matrix	0.1
Rock density	2600 kg/m ³
Rock heat capacity	1 kJ/kg °C
Thermal conductivity	2.1 W/m °C
Initial pressure	10 MPa
Initial temperature	200 °C
Injection rate	0.2 kg/s
Injection temperature	100 °C
Productivity index	$1 \times 10^{12} \text{ m}^3$
Production pressure	9.65 MPa

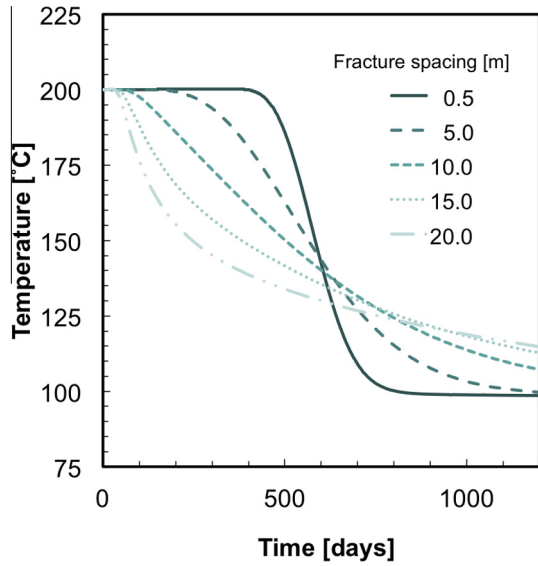


Fig. 1. Effect of fracture spacing on temperature change in the MINC.

(R^2) of the fHTE, the porous model, and the single-fracture model are plotted in Fig. 3. For small fracture spacing (0.5 m), the porous model and the fHTE characterized the temperature history. On the other hand, the temperature history for large fracture spacing (20 m) was fitted well by the single-fracture model and the fHTE. Interestingly, the porous model characterized the result of the

MINC for smaller fracture spacing (<5 m), while the single-fracture model agreed with the results of the MINC for larger fracture spacing (>20 m). In other words, the MINC can reproduce thermal response in intermediate structures between a single fracture and a porous medium by varying the fracture spacings. As shown in Fig. 2, the fHTE was in good agreement with the temperature histories for different fracture spacings. The better performance of fHTE was also supported by its larger coefficient of determination R^2 and smaller RMSE values compared to those of the porous model and the single-fracture model, as shown in Fig. 3.

The correlations between fracture spacing and the best-fit parameters in the fHTE are plotted in Fig. 4. The larger fracture spacing leads to the small retardation parameter e_m and the small index γ . As explained in Section 2.3, in the particular case where the index on time fractional derivative is set to 1/2 (i.e., $\gamma = 1/2$), the solution reduces to the well-documented formulation of Fourier diffusion [16]. The Lagrangian approach determines the index γ in the mean-squared displacement $\langle r \rangle \sim t^\gamma$, instead of the form $\langle r \rangle \sim t^{1/2}$ for normal diffusion. Note that the displacement is proportional to the convective speed with $\gamma = 1$. Therefore, the values of $1/2 < \gamma < 1$ lead to faster (superdispersive) spreading, while the values of $0 < \gamma < 1/2$ cause slower (subdiffusive) spread [45]. Since thermal conduction in the fracture was negligible compared with the convection, the thermal response can be characterized solely by convection process, and the porous model was good agreement with the result for small fracture spacing. The small fracture spacing (≤ 5 m) led to $\gamma = 1$ in the fHTE. When $\gamma = 1$, the coefficient e_m can be considered the thermal retardation factor in the conventional model (e.g., [38]). The simulated time was normalized by the representative time of fluid flow, which was

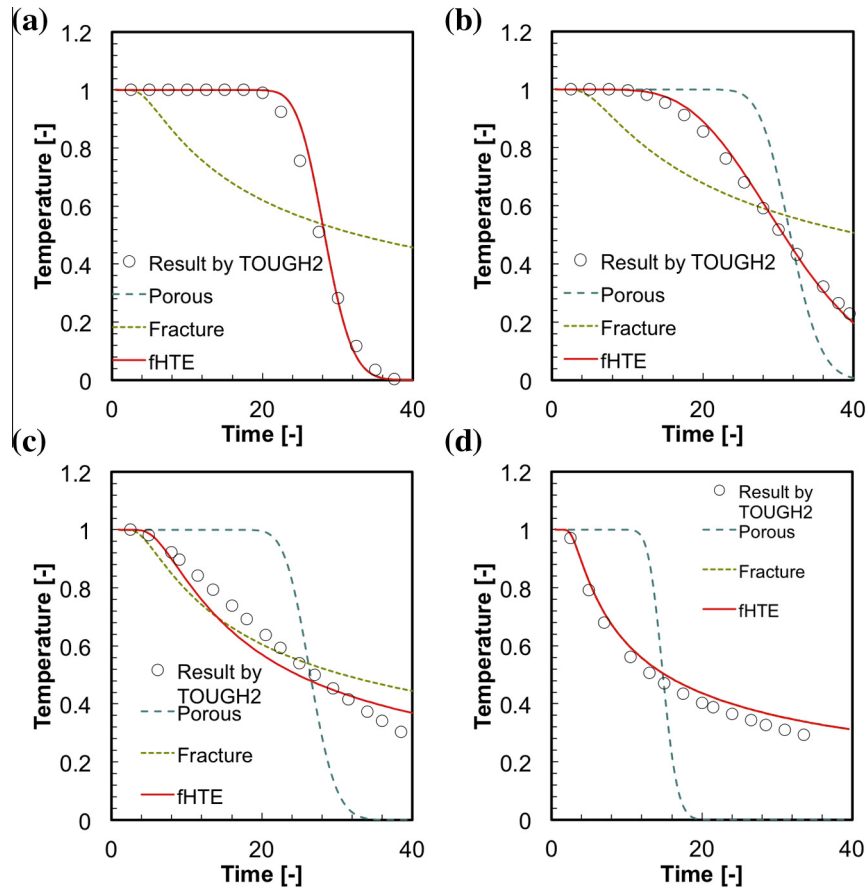


Fig. 2. Best fits with the conventional models and the fHTE for fracture spacing of (a) 0.5 m, (b) 5 m, (c) 10 m, and (d) 20 m.

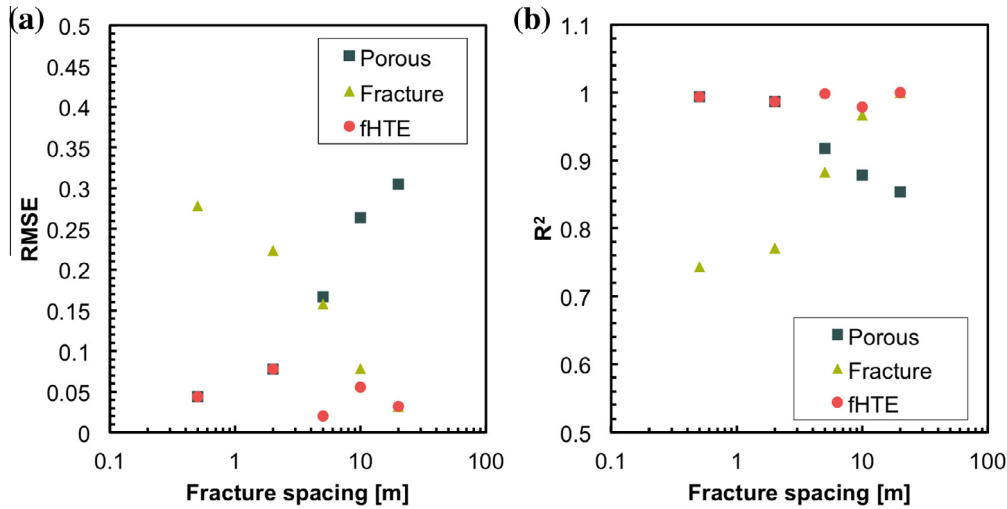


Fig. 3. Accuracy of fitting for the MINC model.

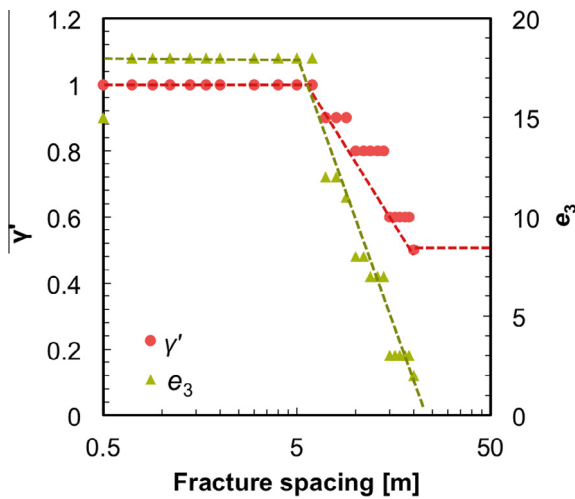


Fig. 4. Best-fit parameters of the fHTE for different fracture spacings in the MINC.

determined by the tracer peak time. The large retardation parameter indicates that the thermal front was delayed from the fluid front.

The large fracture spacing (≥ 20 m) led to $\gamma = 1/2$ in the fHTE. When $\gamma = 1/2$, the thermal diffusion into matrix can be considered Fourier's heat diffusion in the fHTE. This is consistent with the fact that the single-fracture model accounts for heat flux into matrix following the Fourier's law. Large fracture spacing increases in fracture aperture and flow rate in the fracture. Injected water through the fracture reached at the production point rapidly as well as the thermal front. This led to decrease in e_m . Intermediate fracture spacings in the MINC yielded anomalous thermal behaviors, which cannot be described by the porous model and the single-fracture model. The order of fractional derivative was determined to $1/2 < \gamma < 1$, which suggests the migration process is superdispersive spreading.

The effect of heat flux into surrounding rocks by using fracture distribution at damage zones, which was also taken account into the fHTE (22), was investigated in Suzuki et al. [40]. They simulated temperature histories due to anomalous diffusion into the surrounding rocks. The details of the numerical simulations can be found in [40]. The permeability of surrounding rock is given by $K(y) = K_0 y^{-\theta}$ where K_0 is the constant permeability of the main conduit and y is the distance from the main conduit, which is

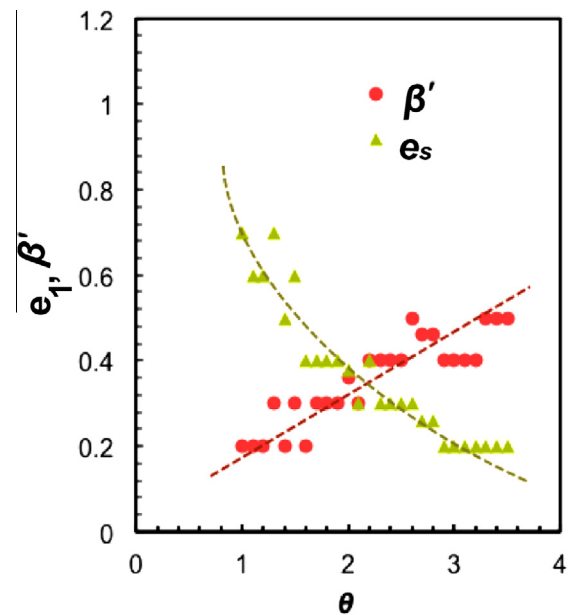


Fig. 5. Best-fit parameters of the fHTE for different decay rates in permeability at damage zone.

perpendicular to the main flow (x -axis). The parameter θ describes the decline rate of permeability distribution in the surrounding rocks. The correlations between θ and the best-fit parameters in the fHTE are plotted in Fig. 5. Decrease in θ , which caused higher penetration into the surrounding rocks, led to increase in the retardation parameter e_s and decrease in the index β , respectively. According to thermal diffusion into surrounding rocks as shown in Fig. 5, the optimized values of β were $0 < \beta \leq 1/2$. This result suggests that subdiffusion (slow spreading) occurred into the surrounding rocks. The permeability decreases rapidly for larger θ , while the permeability changes slowly for smaller θ . When θ was large, water does not flow into the surrounding rocks and, heat exchange occurred at only the interface between the main flow domain and the surrounding rock. This is consistent with the domain assumed in the single-fracture model. The optimized value of β was close to $1/2$. This indicates that the process can be described by the Fourier's law, which is consistent with the single-fracture model. The value of e_s approached 0, which

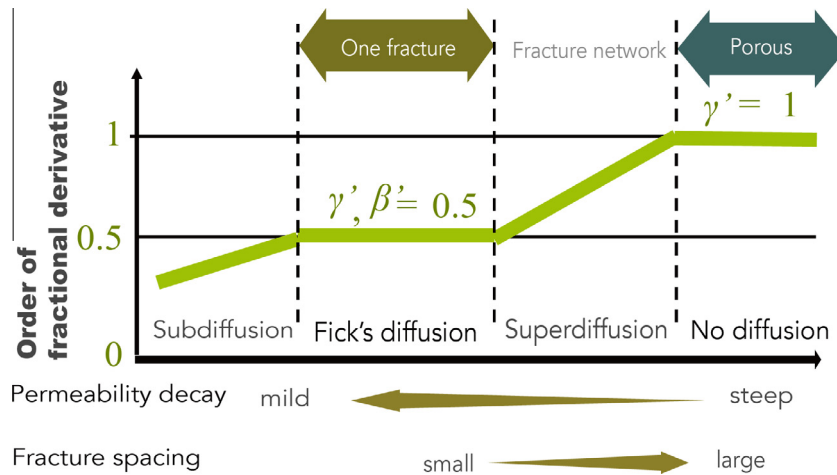


Fig. 6. Meaning of time fractional derivatives.

suggests that the effect of thermal diffusion into the surrounding rock was small. For smaller θ higher permeability area was spread far away from the main conduit. The water and heat front migrated bidimensionally. Because the heat exchange occurred more than the case of a single fracture, the temperature declined slowly on appearance.

According to the comparison with numerical results by the MINC method, we conclude that the fhTE can cover from subdiffusion ($0 < \beta < 1/2$), Fickian diffusion ($\beta, \gamma = 1/2$), and the super diffusion spreading ($1/2 < \gamma < 1$). The relationship between the indexes of time fractional derivative (β and γ) and the fracture structures is summarized in Fig. 6. The fhTE (22) treats two different orders of fractional derivative. The values of the order, β and γ , were determined in different ranges when we simulated different diffusion processes. This suggests that both parameters can be considered separately. Although the parameters use same mathematical expressions, the derivations for each diffusion process were different as introduced in Section 2. The value of fractional-order determined by observation data (γ) can be used to distinguish the diffusion process and estimate the structure in the geological sites. Note that the correlations shown in Figs. 4 and 5 were obtained with uniform fracture aperture and constant rock properties (i.e., heat conductivity, heat capacity, rock density, and permeability). The roughness of fracture surface and heterogeneity of rock properties may affect tailing behavior of heat transfer as well. Because both diffusions into matrix and into surrounding rocks may occur in actual fields, the distributed-order fractional model is effective. The magnitude of retardation parameters would provide the dominant diffusion process.

The governing equation of the fhTE (22) is a macroscopic equation, which treats uniform properties and state variables in an entire reservoir. Classical continuum methods representing state variables in a representative element (i.e., representative elementary volume (REV)) mostly fail to capture fracture–matrix interactions. This is because it assumes that fracture–matrix flow occurs instantaneously and the status variables are uniform in each element. On the other hand, the fhTE accounts for the gradient of temperature in matrix by using time fractional derivatives and can describe the graduation of temperature in matrix. The advantages of using the fhTE in place of the MINC method is shorter computational time to determine constitutional parameters and higher calculation accuracy. The MINC uses limited sub-continua for each grid, which leads to large errors for numerical discretization.

Berkowitz and Scher [3] proposed the CTRW approach, which considers a stochastic jump process with arbitrary distributions of jump lengths and waiting times. A connection between CTRWs

and diffusion equations with time fractional derivatives has been established [36]. Emmanuel and Berkowitz [10] adapted the CTRW framework for heat transfer, and Geiger and Emmanuel [17] investigated non-Fourier thermal transport in fractured media. They concluded that poorly connected fracture pattern is highly non-Fourier and the CTRW can model the heat transfer successfully. In our simulation, larger fracture spacing in the MINC can be explained in the same way of their poorly connected fracture pattern. The case where the parameter β in their model is low reflects the highly non-Fourier nature of thermal transport for those simulations [10]. Their results are consistent with our measured indexes β and γ in this paper.

The analytical solution in Eq. (24) can be obtained for continuous injection ($T_{in} = const.$), while numerical methods to solve the fhTE will provide various injection conditions. Recently, higher order methods that are accurate and fast accuracy and fast [22,39]. The fhTE will provide a powerful tool to characterize thermal response at early stage of geological development.

4. Conclusion

In this paper, we discussed modeling of heat transfer in a fractured reservoir. The fractional heat transfer equation (fhTE) accounts for non-Fourier diffusion by using time fractional derivatives.

Numerical simulation by using the MINC in TOUGH2 generated a simulated thermal response in a fracture medium. Our results revealed that the MINC generates thermal behaviors in between a porous medium and a single fracture by varying fracture spacings. Although conventional analytical models characterize each behavior in either a single fracture or a porous medium, the fhTE can be used to capture intermediate thermal diffusion processes by using time fractional derivative. Comparison with numerical results suggested that the parameters (i.e., the orders of fractional derivative and retardation parameters) in the fhTE correlate with the fracture spacings, which is expected to provide a general guidance for model selection. In natural geological sites where we obtain only sparse and limited observations, the transport process can be characterized efficiently by the fhTE.

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