Modeling Mass and Heat Transfer in Geothermal Reservoirs Using Fractional Differential Equations

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ABSTRACT

Mass and heat transfer equations based on fractional derivatives have been proposed to evaluate the effect of cold water injection into a geothermal reservoir. Numerical simulations are conducted to investigate tracer responses and temperature profiles due to dispersion into the surrounding rocks with varying permeability distribution. The calculated tracer responses exhibit apparent long tails, which are characterized by the mass transport model based on a fractional derivative. The heat transfer model using the fractional derivative is also in a good agreement with the calculated temperature profile.

1. Introduction

A major problem for geothermal reservoir engineers is to prevent thermal breakthrough due to water injection in a geothermal reservoir. Tracer tests have been used to characterize flow of injected water and can provide useful information for design injection operation. In order to represent tracer responses obtained from complex fractured reservoirs, a mass transport model, which is called fractional advection dispersion equation (fADE), has been developed [1]. The advantage of using fADE is its ability to describe fluid flow conditions in a geothermal field with great spatial and temporal heterogeneities, based on a mathematical description of the fundamental physical processes in the reservoir. It is reasonable to suppose that a heat transfer model based on a fractional derivative could also describe the effect of heat transfer into the surrounding rocks. This paper shows the applicability of fADE and and the heat transfer equation based on fractional derivatives to evaluate the effect of water injection into a complex reservoir.

2. Mathematical model

We consider a fractured reservoir as illustrated in Fig. 1. Detailed descriptions to derive the mass transport model are available in Fomin et al. [1]. The fractional advection dispersion equation (fADE) and fractional heat transfer equation (fHTE) can be described as follows:

$$\frac{\partial C}{\partial \tau} + b_3 \frac{\partial^{\gamma} C}{\partial \tau^{\gamma}} + b_1 \frac{\partial^{\beta} C}{\partial \tau^{\beta}} = \frac{1}{Pe}\frac{\partial}{\partial X}\left(p\frac{\partial^{\alpha} C}{\partial X^{\alpha}} + (1-p)\frac{\partial^{\alpha} C}{\partial (-X)^{\alpha}} \right) - \frac{\partial C}{\partial X} \qquad (1)$$

$$\frac{\partial \mathrm{T}}{\partial \tau_h} + e_3 \frac{\partial^{\gamma'} \mathrm{T}}{\partial {\tau_h}^{\gamma'}} + e_1 \frac{\partial^{\beta'} \mathrm{T}}{\partial {\tau_h}^{\beta'}} = -\frac{\partial \mathrm{T}}{\partial X} \qquad (2)$$

where C, T, and X are concentration, temperature and distance, which are normalized with respect to each representative value. The representative concentration is the injected concentration at the inlet point. τ and τ_h are representative time of mass transport and heat transfer, respectively. The velocity in Eq. (1) makes a correlation

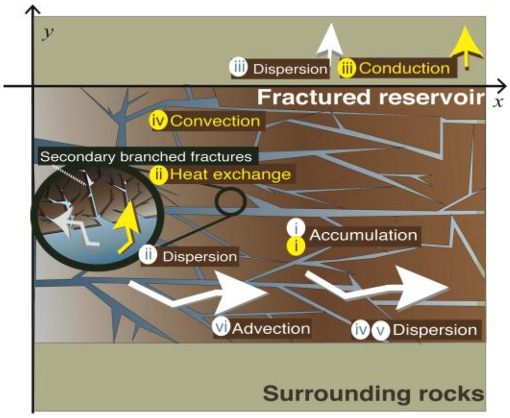

Fig. 1 Schematic of a fractured reservoir

between the representative time and distance. b_1, b_3, e_1, and e_3 are the retardation coefficient and Pe is Péclet number. p ($0 \le p \le 1$) the skew parameter that controls the bias of the dispersion. α ($0 < \alpha \le 1$) is the order of fractional spatial derivative. β, β', γ, and γ' are the order of fractional temporal derivatives, values of which are between 1/2 and 1. Here, the first terms on the left side of Eqs. (1) and (2) are the accumulation term, and the second terms on the left side of Eqs. (1) and (2) model the retardation processes associated with dispersion into secondary branched fractures with respect to mass and heat transfer, respectively. The third term on the left side is the process of vertical dispersion into surrounding rock masses with respect to mass and heat transfer, respectively. The first and second terms on the right side of Eq. (1) express dispersion within the fractured reservoir, and the third term is the advection term. The right side term of Eq. (2) represents convection term. In this study, a finite difference approach is used to solve the Eqs. (1) and (2) [2].

3. Numerical simulation methods

A reservoir simulator TOUGH2 [3] was used to evaluate the effect of cold water injection on reservoir performance. Fluid, mass, and heat flow are numerically simulated in a two-dimensional reservoir model. The numerical properties are summarized in Table 1.

The injection takes place along the entire left side of the domain, and the extraction is performed on the right side. The tracer is injected at 0.2 kg/s for one day, after which the injection switches to fresh water.

The non-Fickian behavior occurs due to the geological structures and dispersivities are considered to be fractal [1]. In this study, the permeability of the surrounding rock, K_s is defined as a function of power law.

4. Results and Discussion

Numerical tracer results obtained using TOUGH2 shows the dependence of the permeability of the surrounding rocks. When the flow was controlled only by advection in the reservoir, the tracer response exhibited a Gaussian curve. In contrast, a response curve for a spatially varying permeability distribution of the surrounding rocks produced a heavy tail and gradual decrease in breakthrough curve.

In the case of impermeable surrounding rocks, both fADE and classical ADE were in a good agreement with the tracer response. The long tail behavior in the tracer response of permeable surrounding rocks can be calculated by fADE, the third term on the left hand side of which expresses the effect of dispersion into surrounding rocks. Figure 2 shows the best-fit of fADE onto the calculated tracer responses for spatially varying permeability of the surrounding rocks. The higher permeability of surrounding rocks resulted in an increase in retardation parameter in fADE as shown in Table 2. For the constant permeability of surrounding rocks, the order of temporal fractional derivative in fADE was shown to be constant. On the other hand, for a spatially varying permeability, the order was dependent on the change in permeability.

The solutions of the fHTE are in a good agreement with the calculated temperature profiles as shown in Fig. 3. The curve fittings on the temperature profile using the fHTE suggested that the order of the fractional derivative in the heat transfer equation was close to that in fADE (Table 2). However, the retardation factor in the heat transfer model differed from the factor in the fADE.

5. Concluding remarks

The simulation of water injection into fractured reservoirs revealed the interplay of different permeability distributions in the surrounding rocks. The calculated tracer responses for the permeable surrounding rocks produced a heavy tail, which can be characterized by the fADE solution. The fADE parameters show relations with the permeability of surrounding rocks. The curve fittings on the temperature profile using the fHTE suggested that the order of the fractional derivative in fHTE was close to that in fADE. However, the retardation factor in the heat transfer model differed from the factor in the fADE.

References

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[2] M. Meerschaert and C. Tadjeran, Appl. Numer. Math, **56** (2006), 80-90.

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Table 1 Summary of the reservoir and numerical properties

Property	
Calculation domain	80 m × 20 m
Element size	1m × 1m × 1m
Thickness of reservoir	2m
Permeabilities:	
Reservoir	1.0×10^{-13} m^2
Surrounding rock	$0 - 1 \times 10^{-14}$ m^2
Porosity	0.1
Rock density	2600 kg/m^3
Rock heat capacity	1 kJ/kg°C
Thermal conductivity	0 W/m°C
Initial pressure	10 MPa
Initial temperature	175°C
Injection temperature	35°C
Productivity Index	1×10^{-8}m^3
Production pressure	9 MPa

Table 2 Estimated parameters of ADE, fADE and fHTE for spatially varying permeability of the surrounding rocks $Ks(y) = y^{-\theta} \times 10^{-13}$

Ks	ADE	fADE		fHTE	
	b_1	b_1	β	e_1	B'
1.5	0.7	1.5	0.1	2.9	0.1
1.8	0.5	0.8	0.2	2.3	0.2
2	0.2	0.3	0.3	1.9	0.5
2.5	0.1	0	1.0	1.6	0.7

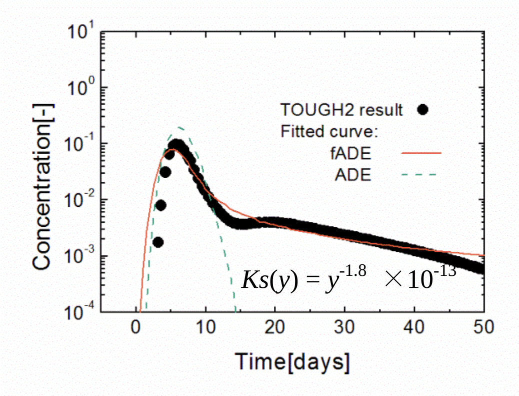

Fig. 2 Simulated tracer responses by TOUGH2 and best fits with fADE and ADE

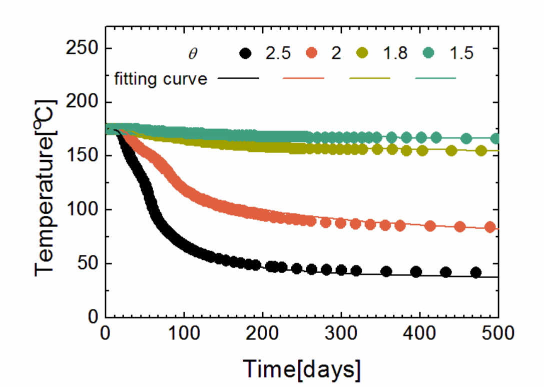

Fig. 3 Simulated temperature profiles using TOUGH2 and the fitting curve of fHTE