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DEVELOPMENT OF FRACTIONAL DERIVATIVE-BASED MASS AND HEAT TRANSPORT MODEL

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ABSTRACT

A numerical scheme to evaluate the effect of cold water injection in a geothermal reservoir has been proposed. The governing equations are derived based on non-Fickian mass transport model. We assume conduction into surrounding rocks using fractional derivative in time, as well as non-Fickian diffusion into the surrounding rocks. The constitutive parameters in the heat transfer model are able to determine mechanisms of heat-fluid–rock interaction within the reservoir of a complex geological structure.

Numerical investigations have shown that topography of a reservoir wall affects tracer responses and temperature distributions. The long tails can be found in tracer response curves in a reservoir with rough surfaces. The permeability difference between a reservoir and surrounding rocks leads to retardation of tracer and thermal breakthrough.

INTRODUCTION

The life of geothermal resources may be prolonged by a reinjection process, which can avoid pressure decline and prevent depletion of water. One of important problems with reinjection is, however, the possibility of an early thermal breakthrough in production wells. Premature breakthrough and injection-induced cooling still continue to be a problem associated with injection into geothermal reservoirs.

Tracer testing is a standard method for tracing mass transport within a geothermal reservoir and can be a valuable tool in the design and management of production and injection operations. The advection-dispersion equation (ADE) based on Fick’s law has been used widely to solve a range of problems in analyzing mass transport. Nevertheless, Fickian solution shows a disagreement in highly fractured reservoirs. In contrast, the concept of fractional derivatives has been applied recently to mass transport model, called fractional advection-dispersion equation (fADE) (Benson et al., 2000). Through the use of fractional derivatives in time and space, Fomin et al. (2005) derived an equation accounting for the effects of tracer transport retardation caused by non-Fickian diffusion into the confining rock. The model allows us to reproduce tracer responses including heavy tails, which are observed often in geothermal fields.

Other means of analyzing tracer testing have appeared in the literature. The chemical front is closest to the thermal front in transport properties, and because it arrives earlier can be a useful precursor of thermal breakthrough. Several authors (Bodvarsson, 1972; Kocabas, 2004) have shown analytical solution of temperature in a fracture corresponding to single-phase liquid flow in the fracture with heat conduction from a semi-infinite matrix, and the solution is also given in Carslaw and Jaeger (1959).

Previous studies derived the equations for flow between smooth, parallel plates. These models, however, show disagreement in data from a highly fractured reservoir. Because the fADE model can describe mass transport interaction between a reservoir and the surrounding formation, it is reasonable to support that the effect of heat conduction into surrounding rocks can be described with fractional derivative model.

The purpose of this paper, therefore, is to propose a new method to evaluate the effect of cold water injection in a complex reservoir using a heat conduction equation based on fractional- derivatives. We focus on condcuction into surrounding rocks and investigate the effect on -thermal breakthrough due to water injection.

MATHEMATICAL MODEL

We consider a fractured reservoir, as illustrated in Figure 1, with following assumptions:

1. Fractured aquifer consists of porous blocks and fractures.
2. The fracture distributions are fractal.
3. Contaminants migrate due to advection and dispersion within fractured aquifer.
3. Thermal transport within fractured aquifer in the X direction is assumed to occur by advection along the fractures; heat conduction is ignored.
4. Mass exchange between fractures and porous blocks arises from dispersion into secondary fractures, whereas heat exchange between them occurs by the dispersion and is regarded as heat conduction into secondary fractures.
5. Dispersion and conduction in the Y direction is assumed to be infinite.
6. The both values of concentration and temperature on the boundary of the fractured aquifer correspond with the values of them on the interface between the reservoir and surrounding rocks.

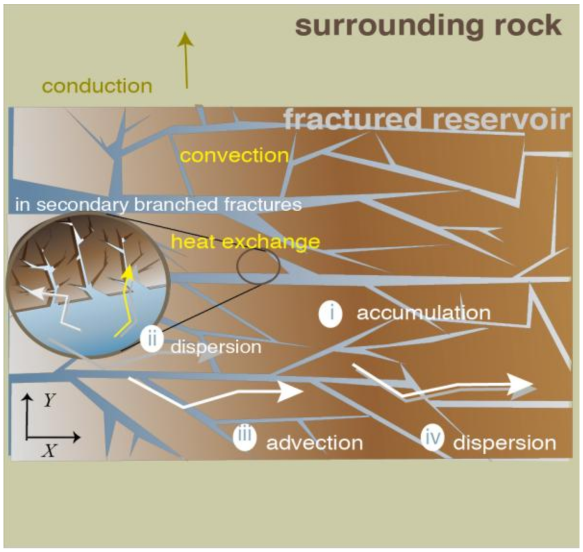

Figure 1: Schematic of a reservoir.

Detailed descriptions to derive the mass transport model are available in Fomin et al. (2005, 2011). The mass transport equation can be described as follows:

$$\frac{\partial m^{(2)} c_2}{\partial t} + a_3 \frac{\partial^\gamma c_2}{\partial t^\gamma} + a_1 \frac{\partial^\beta c_2}{\partial t^\beta} = -\frac{\partial}{\partial x}(m^{(2)} J_2) - v\frac{\partial c_2}{\partial x} \quad (1)$$

$$J_2 = D_2 \left(p\frac{\partial^\alpha c_2}{\partial x^\alpha} + (1-p)\frac{\partial^\alpha c_2}{\partial(-x)^\alpha} \right) \quad (2)$$

where t is time. i=1, i=2 and i=3 denote the properties of the surrounding rocks, of the fractured reservoir, and of the porous blocks, respectively. c_i and J_i are the mean concentration and the mean diffusive mass flux in the fractured reservoir in the x-direction of i-zone, respectively. m_i, D_i, and a_i are the porosity, the effective diffusivity and retardation factor of mass transport process of i-zone, respectively. v is the mean velocity, with which the flow of solutions takes place only along fractures. α ($0< \alpha \leq 1$), β ($0< \beta < 1$) and γ ($0.5 \leq \gamma \leq 1$) are the order of fractional spatial and temporal derivatives, respectively. p ($0 \leq p \leq 1$) is the skewed parameter that control the bias of the dispersion(Huang et al., 2008). The parameter p reflects the relative weight of forward versus backward transition probability. If p is smaller than 1/2, the dispersion is skewed backward describing a slowly evolving contaminant plume followed by a heavy tail. If p is greater than 1/2, the dispersion is skewed forward describing a fast evolving contaminant plume followed by a light tail. If the solute forward transition has the same probability as the backward transition, p equals to 1/2. This is the symmetric case for the solute transport. In this study, we set p to 1/2.

We assume that the heat transfer equation can be derived in the same way of mass transport equation, which is described as follows:

$$\overline{\rho C_{p2}} \frac{\partial T_2}{\partial t} + d_3 \frac{\partial^{\gamma'} T_2}{\partial t^{\gamma'}} + d_1 \frac{\partial^{\beta'} T_2}{\partial t^{\beta'}} = -\rho_w C_{pw} u_w \frac{\partial T_2}{\partial x} \quad (3)$$

where $\overline{\rho C_{p2}} = m^{(2)} \rho_w C_{pw2} + (1 - m^{(2)}) \rho_r C_{pr2}$. T is the temperature. ρ_j and C_{pw} are the density and heat capacity of water (j=w) and of rock (j=r), respectively. d_i means a heat loss factor (a heat retardation factor) caused by heat transfer processes of i-zones. β' ($0< \beta' < 1$) and γ' ($0.5 \leq \gamma' \leq 1$) are the order of fractional derivatives, respectively. It is interesting to reveal whether the order of fractional derivatives β' and γ' in heat conduction equation (3) relate to the order of fractional derivatives β and γ in mass transport equation (3). Further consideration will be needed to yield any findings about the relationship.

In order to convert equations (1)-(3) into non-dimensional form, the characteristic scales must be defined. The scale for time of mass transport, t_m and the scale for time of heat transport, t_h represent the characteristic time for contaminant and heat migrate the representative length, l. The non-dimensional variables can be introduced as follows:

$$X = \frac{x}{l};\ C = \frac{c_2}{c_m};\ \tau = \frac{t}{t_m};\ \Theta = \frac{T_2}{T_h}; \tau_h = \frac{t}{t_h};$$
$$Pe = \frac{x_m^{1+\alpha}}{t_m D_2}; b_1 = \frac{a_1}{m^{(2)} t_m^{\beta-1}};\ b_3 = \frac{a_3}{m^{(2)} t_m^{\gamma-1}}; \quad (4)$$
$$e_1 = \frac{d_1}{m^{(2)} t_h^{\beta-1}};\ e_3 = \frac{d_3}{m^{(2)} t_h^{\gamma-1}};$$

Substituting the non-dimensional variables from equation (4) into equation (1) and (2) yields non-dimensional mass transport equation as follows:

$$\frac{\partial C}{\partial \tau} + b_3 \frac{\partial^\gamma C}{\partial \tau^\gamma} + b_1 \frac{\partial^\beta C}{\partial \tau^\beta} = \frac{1}{Pe}\frac{\partial}{\partial X}\left(p\frac{\partial^\alpha C}{\partial X^\alpha} + (1-p)\frac{\partial^\alpha C}{\partial (-X)^\alpha}\right) - \frac{\partial C}{\partial X} \quad (5)$$

We would like to focus attention on the third term of left-hand side in equation (5). The term allows us to describe non-Fickian diffusion process into surrounding rock mass.

As well as mass transport equation (5), heat conductive equation can be driven as follows:

$$\frac{\partial \Theta}{\partial \tau_h} + e_3 \frac{\partial^{\gamma'} \Theta}{\partial \tau_h^{\gamma'}} + e_1 \frac{\partial^{\beta'} \Theta}{\partial \tau_h^{\beta'}} = -\frac{\partial \Theta}{\partial X} \quad (6)$$

The most noteworthy point is that the third term in left-hand side is supposed to describe heat conduction into the surrounding rocks.

We present a finite-difference approach to solve the equations (5) and (6). It is assumed that $0 \le C(X, \tau)$ over the region $0 \le X \le L$, $0 \le \tau \le \tau_{max}$. Define $\tau_n = n\Delta\tau$ to be the integration time $0 \le \tau_n \le \tau_{max}$, $\Delta x > 0$ to be a grid size in spatial dimension where $\Delta X = L / NX$, $X_i = i\Delta X$ for $i = 0, ..., NX$ so that $0 \le X_i \le L$. Let an approximation to $C(X_i, \tau_n)$. We have the finite-difference solution combining equations (5) and (6), which are discretized in time using an implicit (Euler) method:

$$\frac{C_i^{n+1} - C_i^n}{\Delta \tau} + \frac{b_3}{\Delta \tau^\gamma}\sum_{k=0}^{n} w_k^\gamma C_i^{n+k-1} + \frac{b_1}{\Delta \tau^\beta}\sum_{k=0}^{n} w_k^\beta C_i^{n+k-1} = \frac{J_{i+1}^{n+1} - J_i^{n+1}}{Pe\Delta X} - \frac{C_{i+1}^{n+1} - C_{i-1}^{n+1}}{2\Delta X} \quad (7)$$

$$\frac{\Theta_i^{n+1} - \Theta_i^n}{\Delta \tau_h} + \frac{e_3}{\Delta \tau_h^{\gamma'}}\sum_{k=0}^{n} w_k^{\gamma'} \Theta_i^{n+k-1} + \frac{e_1}{\Delta \tau_h^{\beta'}}\sum_{k=0}^{n} w_k^{\beta'} \Theta_i^{n+k-1} = -\frac{\Theta_{i+1}^{n+1} - \Theta_{i-1}^{n+1}}{2\Delta \tau_h} \quad (8)$$

In this work, we use the prescribed-flux boundary, which has a prescribed flux at the inlet $X = 0$ and a free drainage at the outlet $X = L$ (Zhang et al., 2007).

TRACER RESPONSE AND THERMAL BREAKTHROUGH CORRESPONDING TO CONDUCTION INTO A SURROUNDING ROCK

In this study, we limit the discussion to the effect of dispersion and heat conduction into surrounding rocks, which expresses the third term in left hand side in equations (5) and (6), and not take up the second term. Tracer response curves were simulated, as shown in Figure 3, for an instantaneous source input with $Pe = 10$, $\alpha = 1.0$, $b_3 = 0$, and $p = 1/2$, respectively. Figure 2(a) shows the effect of the retardation factor of b_1 values, which set to 0.1, 0.4, 0.7 and 1.0 at $\beta = 0.3$. The peak concentration of pulse tends to decrease with large values of b_1, and then, the response curve exhibits heavy tail. Figure 2(b) shows the effect of the fractional derivative β values, which set to 0.1, 0.3 and 0.5 at $b_1 = 0.4$. The peak concentration does not depend on β, whereas the response curve includes heavy tail with smaller values of β. The smaller values of β are known to simulate solute diffusion into the surrounding rock in greater volumes. This leads to greater retardation of contaminant transport in the reservoir and gives rise to heavy tail in the tracer response curve.

Simulated temperature profiles for a continuous source are shown in Figure 3. Figure 3(a) shows the effect of the retardation factor of e_1 values, which set to 0.1, 0.4, 0.7 and 1.0 at $\beta' = 0.3$. Thermal breakthrough tends to be slower with large values of e_1. In contrast, Figure 2(b) shows the effect of the fractional derivative β' values, which set to 0.1, 0.3 and 0.5 at $b_1 = 0.4$. The gradient of thermal breakthrough does not depend on β', whereas temperature for long period increases with smaller values of β'. It suggests that even if thermal breakthrough occurs earlier, temperature does not always decline. However, the conventional model without fractional derivative may overestimate the temperature drop due to cold-water injection.

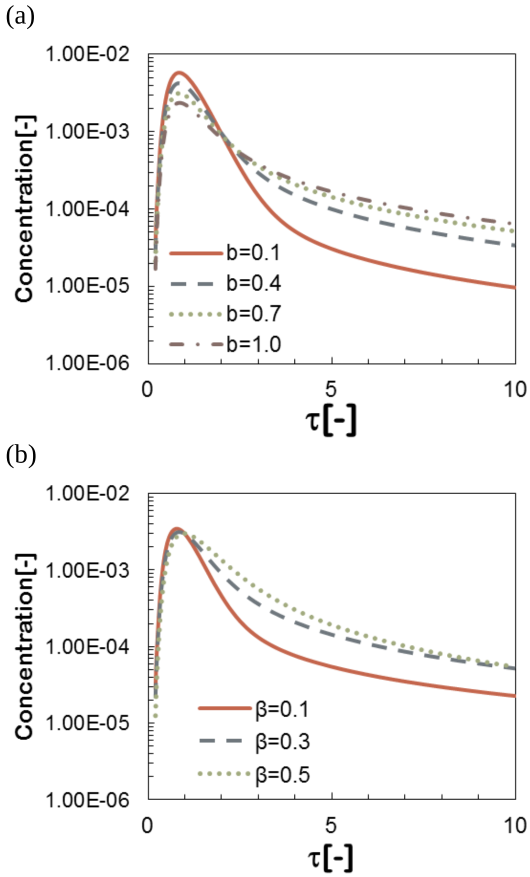

Figure 2. Tracer responses corresponding to dispersion into a surrounding rock.

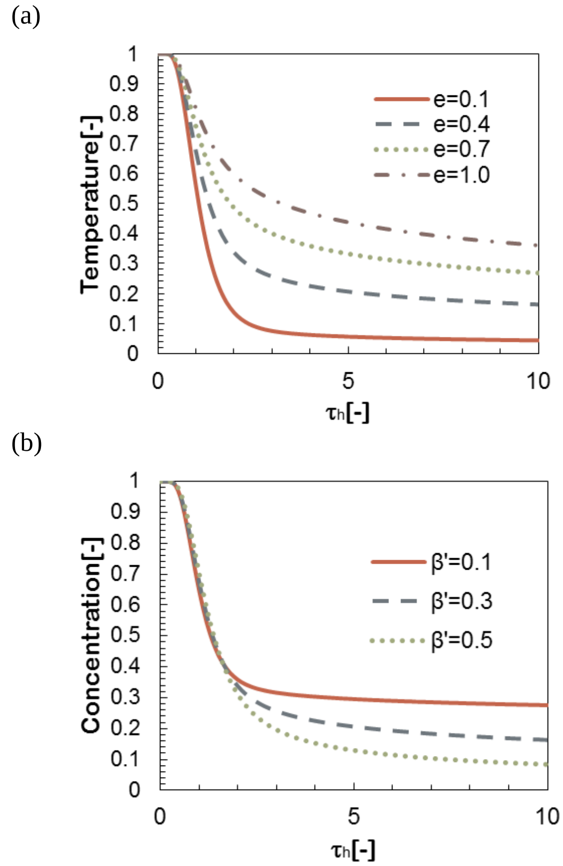

Figure 3. Temperature profiles corresponding to conduction into a surrounding rock. Results and Discussion

EFFECT OF COLD-WATER INJECTION

Numerical Simulation

As discussed in the introduction, we are concerned with the effect of cold-water injection on a reservoir behavior, such as tracer and thermal breakthrough. Given this objective, finite injection tracer test in a vertical, two- dimensional, homogeneous permeable medium are simulated using TOUGH2 geothermal reservoir simulator. The numerical properties are summarized in Table 1. Injection occurs along the entire left side of the domain, and extraction is from the right. Tracer is injected at 0.3 kg/s for 1 day at first, and then switches to fresh water. Initial pressure and temperature were taken as 10 MPa and 300℃, respectively. Injection temperature was 100 ℃. Production occurred against a downhole pressure of 9.5 MPa. The boundary condition is no flow. We consider a permeable reservoir with high permeability ($1x10^{-13}$ m^2) and relative impermeable surrounding rocks above and below the reservoir with low permeability ($1x10^{-16}$ m^2). The boundary between reservoir and surrounding rock is considered to be rough (see Figure 4).

Tracer response and temperature histories are shown in Figure 6. In the case where the permeability of the surrounding rock is set to zero, the boundary condition is considered to be Direchlet type. In contrast, if the permeability of surrounding rock is given a relatively high value, the top and bottom elements are described by Neumann no-flow conditions because assigning very large volumes to such boundary elements will ensure that their thermodynamic state remains unchanged in a simulation.

The time of concentration peak does not depend on the permeability of surrounding rocks. In contrast, a second peak appears in the breakthrough curve with higher permeability of surrounding rocks. A flow domain is composed of two permeable zones in the same way of a dual porosity/permeability model. Since tracer migrates into the surrounding rock, the value of peak concentration is smaller than with lower permeability of surrounding rock.

Table 1 Summary of reservoir and numerical properties.

Property	
Reservoir properties	
Thickness	1000 m
Well spacing (distance from injector to producer)	1000 m
Permeability	$1.0x10^{-13}$ m^2
-Permeable zone	$5.0x10^{-15}$~$1.0x10^{-16}$ m^2
-Surrounding rock	0.1
Porosity	1 kJ/kg℃
Rock heat capacity	
Thermal conductivity	0 W/m℃
Initial conditions	
Pressure	10 M Pa
Temperature	
Injection rate	300 ℃
Injection temperature	0.3 kg/s
Production pressure	100 ℃
	9.5 MPa

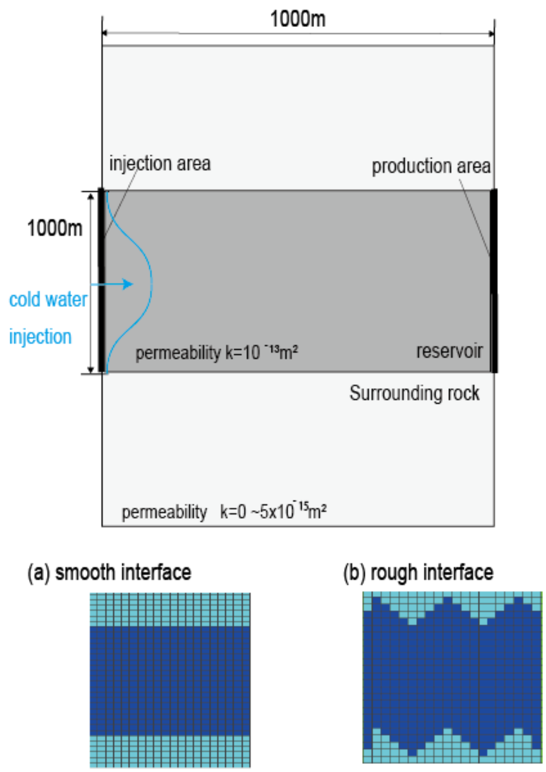

Figure 5: Simulation model to evaluate the effect of cold water injection

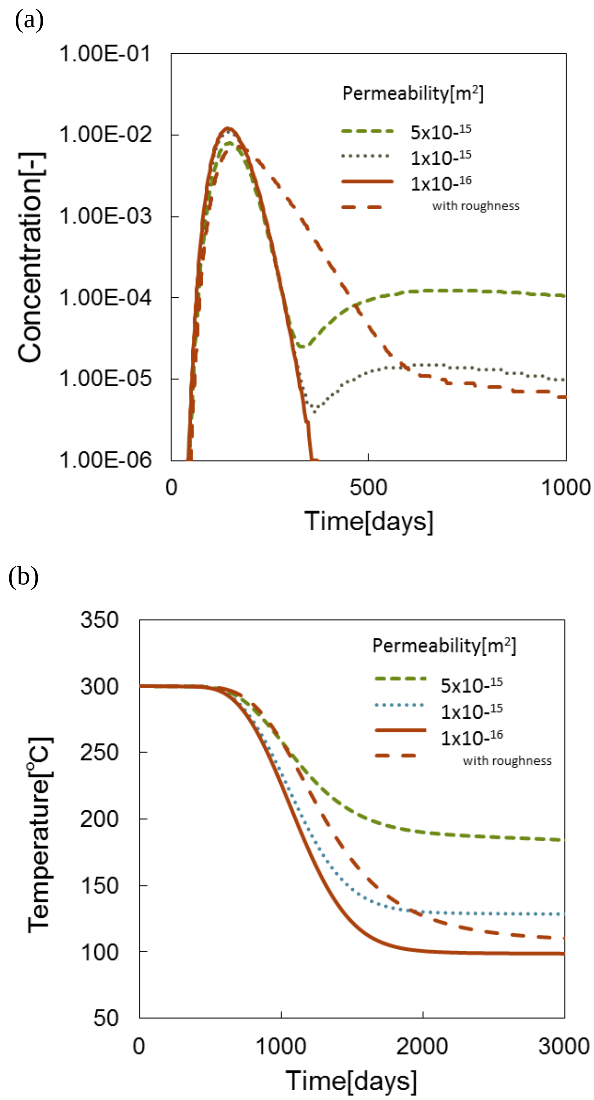

Figure 6: (a)Tracer response and (b)temperature histories at permeability of $1x10^{-14}m^2$ (gray line) and $1x10^{-16}m^2$ (black line), showing asperity effects.

At a surrounding permeability of $1x10^{-16}m^2$, an apparent second peak was not observed with the smooth boundary. On the other hand, the asperities of the boundary surface lead to long tails in tracer responses. It is clear that the tracer return is affected by the reservoir boundary structure. The size of the asperities may provide different features in the tracer response curves at late time. Although the results require further investigation to compare with the constitutive parameters in the fADE model, the fADE model have been found to reproduce heavy tail in tracer responses. Hence, it is reasonable to suppose that fADE model can characterize the structure of reservoirs including the roughness of surfaces.

As well as tracer response, the temperature histories show that the higher surrounding permeability, the more slowly thermal breakthrough occurs because thermal velocity is related to fluid velocity. In the case where the reservoir boundary was rough, the temperature declined faster temporally at early time. Because forming the roughness in surrounding rock expands the permeable zone, premature advancement of injected cold water could be easier to reach to the production area. After a while, the temperature history in the case with rough surfaces shows slow decline in comparison with smooth surfaces. Here, the temperature distributions provide significant difference between smooth and rough boundaries. For higher surrounding permeability, cold water tends to infiltrate into surrounding rocks along the rough structure. Because the boundary surface which contacts cold water increases with rough boundary, the water extracts heat from surrounding rocks and allows us to delay thermal breakthrough.

It should be emphasized that the Eq. (6) assumed the effect of conduction into surrounding rocks in a complex reservoir. Because the roughness of boundary leads to thermal retardation, the analytical model based on fractional derivatives provides a valuable tool to estimate premature thermal breakthrough. Further studies are required in order to elucidate the relationship between the analytical model and the asperities of the reservoir boundary.

CONCLUDING REMARKS

A mathematical model for anomalous heat transport within a geothermal reservoir has been proposed. The proposed governing equations are based on non-Fickian mass transport model, which include fractional derivatives in the partial differential equation. We describe conduction into surrounding rock using fractional derivatives in time, as well as non-Fickian diffusion in the surrounding rock mass. The finite difference scheme to solve the equation has been developed. The constitutive parameters in the proposed equation are able to determine mechanisms of heat-fluid–rock interaction within the reservoir of a complex geological structure.

Our simulations of cold-water injection in reservoirs reveal interplay of processes for different asperities of the reservoir boundary. The long tails in tracer responses can be found in reservoirs with rough boundary surfaces. The conduction process into surrounding rock leads to retardation of tracer and thermal transport. Future studies are planned to quantify these aspects using anomalous heat transfer equation.

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REFFERENCE

Benson, D. A., Wheatcraft, S. W. and Meerschaert, M. M. (2000), “Application of a fractional advection-dispersion equation,” *Water Resources Research*, **36**(6), 1403–1412.

Bodvarsson, G. (1972), “Thermal problems in siting of reinjection wells,” *Geothermics*, **1**, 2, 63–66.

Carslaw, H.S. and Jaeger, J.C. (1959), “Conduction of Heat in Solids, *second ed., Oxford Science Publications*, 396.

Fomin, S. A., Chugunov, V. A. and Hashida, T. (2005), “The effect of non-Fickan diffusion into surrounding rocks on contaminant transport in a fractured porous aquifer,” *Proc. R. Soc. A*, **461**, 2923-2939.

Fomin, S. A., Chugunov, V. A. and Hashida, T. (2011), “Non-Fickian mass transport in fractured porous media,” *Advances in Water Resources*, **34**(2), 205–214..

Huang, Q., G. Huang, and H. Zhan. (2008), “A finite element solution for the fractional advection–dispersion equation,” *Advances in Water Resources*, **31**(12),1578–1589.

Kocabas, I. (2004), “Geothermal reservoir characterization via thermal injection backflow and interwell tracer testing,” *Geothermics*, **34**(1), 27–46.

Meerschaert, Mark M, and C Tadjeran. (2006), “Finite difference approximations for two-sided space-fractional partial differential equations,” *Applied Numerical Mathematics*, **56,** 80–90.